International Journal of Theoretical Physics, Vol. 45, No. 12, December 2006 (© 2006)
DOI: 10.1007/s10773-006-9186-0

What is "System": Some Decoherence-Theory Arguments

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Received February 16, 2006; accepted March 13, 2006
Published Online: June 6, 2006

We discuss the possibility of making the *initial* definitions of mutually different (possibly interacting, or even entangled) systems in the context of decoherence theory. We point out relativity of the concept of elementary physical system as well as point out complementarity of the different possible divisions of a composite system into "subsystems," thus eventually sharpening the issue of "what is system."

KEY WORDS: decoherence; open systems; quantum statistical methods.

1. INTRODUCTION

A physical system is described by its parameters (e.g., mass, electric charge etc.) and by the degrees of freedom properly describing dynamics of the system. In the classical world, this general scheme seems inevitable. Nevertheless, in the quantum realm, the things may be different as we show within the context of the *"environment-induced superselection rules"* (or *decoherence*) theory.

Actually, the task of dividing complex systems into subsystems is not in general trivial. This fundamental yet a subtle task can be performed in some generality on the basis of the decoherence theory, yet bearing certain open questions. E. g. a composite system $\mathcal{C}$ may not be divisable in respect to the arbitrarily defined "degrees of freedom," thus–relative to *these* degrees of freedom–being an *elementary* physical system (likewise the elementary particles). On the other side, the possible (meaningful) division of $\mathcal{C}$ into subsystems need *not*, in principle, be unique, thus posing the question of physical reality of the "subsystems" emerging from the different possible divisions of $\mathcal{C}$. Bearing in mind that the real systems are usually open systems, the task of defining "subsystem" coincides with the task of defining "system."

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The method employed here is elementary yet conceptually sufficient for addressing the truly fundamental issue of "what is system" within the context of quantum mechanics of open systems.

2. THE PROBLEM

Most of the "quantum paradoxes" start with the assumption of existence of mutually separable physical systems. On the other side, quantum holism removes most of the problems (except on the intuitive level) from the very beginning–being a consequence of the *fully consistent* quantum mechanical formalism (e.g. of the quantum entanglement). In the macroscopic domain, however, existence of the well-defined, mutually separated systems is the very basis of the physical methods and is actually taken for granted. Thus, in a sense, transferring the concept of the different systems from the macroscopic, through the mesoscopic, to the purely quantum-mechanical domain is at the heart of the problem of the "transition from quantum to classical" (Giulini *et al.*, 1986; Zurek, 1991, 1993). This is also a problem of practical importance–since, it seems, that in the *realistic* situations, we are able to distinguish between the different systems (e.g. between the object of measurement and the measurement instrument).

The possibility of defining mutually independent (compatible) degrees of freedom (and their conjugate momenta) and of performing independent ("local") measurements of the observables is a *defining feature* of the different physical systems. The importance of the issue is rather apparent. E.g. according to Zurek (1993): "... [quantum mechanical] problems ... cannot be even posed when we refuse to acknowledge the division of the Universe into separate entities," while "... without the assumption of a preexisting division of the Universe into individual systems the requirement that they have a right to their own states cannot be even formulated." Of course, the rules for defining the preferred states (e.g. the "pointer basis," as well as the "pointer observable" (Zurek, 1991, 1993)) of an open system comes from the foundations of the decoherence theory. On this basis appeared an early draft (Dugić, 1999) of the problem considered here.

This issue should be distinguished from the problem of the loss of individuality of mutually *entangled* systems. Actually, the entangled states refer to the, initially, well-defined systems: the systems (actually subsystems) are usually assumed already to be *defined*, as well as their state spaces, which is the basis for defining the entangled states. So, in this perspective, the task of answering "what is system" is a more fundamental task than investigation of entanglement itself.

Essentially the same problem has recently been addressed e.g. by Zanardi *et al.* (2004) and by Barnum *et al.* (2003) (and the references therein), by considering the different possible operational uses of entanglement in the quantum information issues. While bearing some similarity with our results, the results presented therein are based on the different approaches that are briefly discussed in Section 6. Here, we employ the foundations of the decoherence theory and particularly certain recent results in this regard (Dugić, 1996, 1999, 1997).

3. ON DECOHERENCE

Decoherence theory deals with the open quantum systems. It is therefore natural to seek for an answer to the fundamental problem of "what is system" within this theory. Here, we employ the foundations of the so-called "*environment-induced superselection rules*" theory (Zurek, 1991, 1993), which provides a clear conceptual framework for this purpose. To this end, a short survey of the theory might be useful.

In general, by "decoherence" one may assume the different, sometimes even mutually physically exclusive, processes. Here, we refer to the "environment-induced decoherence" effect (Zurek, 1993) that can be defined as follows.

Definition 3.1. By decoherence, we assume the environment-induced, dynamical appearance of the effective superselection rules for an open quantum system. Decoherence determines the so-called "pointer observable" that brings the classical-like behaviour of the system. An orthonormalized basis, $\{|\phi_n\rangle_1\}$, that is an eigenbasis of the "pointer observable" (the "pointer basis") bears *robustness* as formally defined:

$$\hat{H}_{int}|\phi_n\rangle_1|0\rangle_2 = |\phi_n\rangle_1|\chi_n\rangle_2. \tag{1}$$

In Eq. (1): the open system is the system 1 and the system 2 represents its environment. More generally, the "pointer basis" may be substituted by a set of the only-approximatelly-orthogonal states (a "preferred set of states") for which Eq. (1) is only approximately satisfied.

Equation (1) is substantial in the "macroscopic context" of the decoherence theory (Zurek, 1991, 1993), i.e. for the issue of the "transition from quantum to classical" (Giulini *et al.*, 1986; Zurek, 1991, 1993). It is also of interest in the quantum measurement theory. However, Eq. (1) does not appear substantial for the microscopic systems, for which one expects to maintain their genuine, quantum mechanical nature.

Openess of a system is the very origin of the occurrence of decoherence. Actually, if the interaction in the system $1 + 2$ may be *reduced* to the *external field* e.g. for the system 1, then this system remains an isolated system described by the Schrodinger equation, not yet being subject to the decoherence process.

In effect, decoherence determines the classical-like degrees of freedom (then appearing as the "pointer observables") of the open system, while the degrees of freedom intact by the environment may maintain their genuine quantum-mechanical character.

Most of the operational tasks of the decoherence theory rely on investigation of the characteristics of the interaction in the composite system $1 + 2$–which is the subject of the next section.

4. SEPARABILITY

As implicit in the above quotation of Zurek (cf. Section 2), the initial definitions of the subsystems *make sense if one can* a posteriori *justify these definitions* on the basis of the *occurrence of decoherence*. In other words: non-occurrence of decoherence for a system gives rise to ill-defined system and–in a sense–challenges the initial definition of the system. This is exactly the point which our analysis starts from: what might be physically told about an ill-defined open system?

A detailed analysis (Dugić, 1996, 1997) of the occurrence of decoherence points out the condition of *separability* (cf. Definition 4.1 below) of the *interaction term* of the Hamiltonian as the (effective) necessary condition for the occurrence of decoherence–cf. Appendix A. Investigating the occurrence of decoherence is truly a subtle task (Dugić, 1996, 1997; Paz and Zurek, 1999). E.g., separability of the complete Hamiltonian (of the composite system "system + environment") is sufficient in this regard (Dugić, 1996, 1997). Strong interaction allows the occurrence of decoherence. generally, which still depends on a number of the details in the model of the system (Dugić, 1996; Zurek, 1991, 1993). On the other side, strong interaction is not necessary for the occurrence of decoherence (Paz and Zurek, 1999). Nevertheless, the condition of separability of the interaction Hamiltonian represents an (effective) necessary condition for the occurrence of decoherence (Dugić, 1997)–cf. Appendix A for some details.

Now, the *separability appears as a condition useful for defining the "dividing line"* between the subsystems. Formally, existence of the subsystems is presented (cf. (4.1) below) by the tensor-product symbol, $\otimes$, while assuming the definitions of the subsystems through their–implicitly present–degrees of freedom.

Definition 4.2. A bipartite $(1 + 2)$ system's observable $\hat{A}_{12}$ is of the *separable kind*, if its *general* form

$$\hat{A}_{12} = \sum_i \hat{B}_{1i} \otimes \hat{C}_{2i}, \tag{2}$$

fulfills any of the following, mutually equivalent conditions: (A) Its spectral form reads $\sum_{i,j} a_{ij} \hat{P}_{1i} \otimes \hat{\Pi}_{2j}$, where appear the (orthogonal) projectors onto the Hilbert spaces of the two systems; (B) there exist the two orthonormal bases in the state spaces of the systems, $\{|i\rangle_1\}$, and $\{|\alpha\rangle_2\}$ that diagonalize the observable: ${}_1\langle i|\hat{A}_{12}|j\rangle_1 = 0, \forall i \neq j$, and ${}_2\langle \alpha|\hat{A}_{12}|\beta\rangle_2 = 0, \forall \alpha \neq \beta$; (C) every pair of the observable of the system 1 in Eq. (2) mutually commute, and analogously for the system 2.

A constructive proof of existence of the general form Eq. (2) of a bipartite system's observable is given in (Dugić, 1997). Here, we want to emphasize: the form Eq. (2) is a general form for the bipartite-system's observables, such as the interaction Hamiltonian; i.e. an observable is either of a non-separable, or of the separable kind–the later requiring fulfillment of any of the points (A)–(C) of Definition 4.1.

Therefore, operationally, investigating separability of the interaction Hamiltonian gives rise to both (Dugić, 1996, 1997): (i) to the superselection rules defined by the projectors $\{\hat{P}_{1i}\}$ (when the system 1 is considered as the open system), and (ii) to a definition of the pointer observable and therefore of the possible pointer basis (or of the preferred set of states) of the open system–e.g. the system 1 in our notation). Having in mind that the observables, e.g. $\hat{B}_1$s, are the functions of the degrees of freedom of the system 1, the task of investigating decoherence actually assumes the *initially* well-defined (sub)systems.

So, we introduce the following *operational tool* for addressing the problem at issue: the condition of separability appears as a criterion for defining the "dividing line" between the subsystems of a composite system.

5. QUANTUM RELATIVITY OF "SYSTEM"

Usefulness of separability in the foundations of the decoherence theory bears some subtlety yet. The example of the hydrogen atom is paradigmatic in the following sense. The composite system "hydrogen atom (HA)" is originally defined by the Hamiltonian:

$$\hat{H} = \hat{T}_e \otimes \hat{I}_p + \hat{I}_e \otimes \hat{T}_p + \hat{V}_{Coul}, \tag{3}$$

where the Coulomb interaction, $\hat{V}_{Coul}$, couples the positions of the electron (subscript e) and of the proton (subscript p), bearing obvious notation. Having in mind the definition of separability (Section 4), it is straightforward to prove non-separability of $\hat{H}$ yet separability of the Coulomb interaction.

However, the proper *canonical transformations* of the degrees of freedom give another, *separable form* of $\hat{H}$; even more, each single term is (apparently) of the separable kind:

$$\hat{H} = \hat{T}_{CM} \otimes \hat{I}_R + \hat{I}_{CM} \otimes \hat{T}_R + \hat{I}_{CM} \otimes V_{Coul}(\hat{r}_R), \tag{4}$$

where CM stands for the "center of mass" and R for the "relative particle" system; $r_R \equiv |\vec{r}_e - \vec{r}_p|$.

In the *context* of our considerations, these well-known transformations give rise to the following observation. The composite system HA is decomposable into the pair of the quantum particles (e, p) (cf. Section 6 yet). On the other side, the form Eq. (4) of the Hamiltonian refers to the new, also well known, division of HA: the system now reads "Center of mass + relative particle" $(CM + R)$; certainly, $e + p = \mathcal{C} = CM + R$. Due to the small mass-ratio–$m_e/m_p \ll 1$–it is allowed to "identify" CM with p and R with e. Nevertheless, in general, this identification is not physically reasonable, as we show in the sequel. From this example, we learn: *the choice of the degrees of freedom may redefine the Hamiltonian separability, thus* (cf. Section 4) *directly referring to the issue of putting the dividing line between the (sub)systems.*

Let us first briefly consider the case of totally nonseparable Hamiltonian. That is, we assume that a given Hamiltonian can not be (re)written in a separable form by the use of any (linear) canonical transformations. As to the told in Sections 3 and 4, then one can not define the dividing line between the "subsystems" of the composite system defined by the Hamiltonian considered. Then, it seems we are forced to consider the system *undivisable*, thus resembling the concept of elementarity of the quantum particles. Physically, a definition of the subsystems in this case is artificial, and the measurements of the "subsystems' observables" is nothing but the measurements of the observables of the composite system, not yet interpretable in terms of the observables of the well-defined subsystems.

As a counterexample, let us consider the following possibility. A Hamiltonian is separable *relative* to a set of the "degrees of freedom" (and their conjugate momenta), $(\hat{x}_{Ai}, \hat{p}_{Aj}; \hat{\xi}_{Bm}, \hat{\pi}_{Bn})$, thus defining a division of the composite system as $\mathcal{C} = \mathcal{A} + \mathcal{B}$; by definition, $[\hat{x}_{Ai}, \hat{p}_{Aj}] = \imath\hbar\delta_{ij}$ (and analogously for $\mathcal{B}$), while $[\hat{x}_{Ai}, \hat{\xi}_{Bm}] = 0$ and $[\hat{x}_{Ai}, \hat{\pi}_{Bn}] = 0$ (and analogously for $\hat{p}_A$s). But, suppose that the same Hamiltonian can be rewritten in a separable form relative to another (analogous) set of the "degrees of freedom," $(\hat{X}_{Dp}, \hat{P}_{Dq}; \hat{\zeta}_{E\alpha}, \hat{\Pi}_{E\beta})$, thus giving rise to another possible division of the composite system, $\mathcal{C} = \mathcal{D} + \mathcal{E}$. By the assumption: the two sets of the observables are mutually related by the linear canonical transformations

$$\hat{\zeta}_{E\alpha} = f_\alpha(\hat{x}_{Ai}, \hat{p}_{Aj}; \hat{\xi}_{Bm}, \hat{\pi}_{Bn}), \qquad \hat{\Pi}_{E\beta} = g_\beta(\hat{x}_{Ai}, \hat{p}_{Aj}; \hat{\xi}_{Bm}, \hat{\pi}_{Bn}), \tag{5}$$

and analogously for the subsystem $\mathcal{D}$, while assuming the inverse is also defined. Now, the supposed separable forms of the same interaction Hamiltonian read:

$$\hat{H}_{int} = \sum_m C_m(\hat{x}_{Aj}, \hat{p}_{Aj}) \otimes D_m(\hat{\xi}_{Bl}, \hat{\pi}_{Bl}) \tag{6}$$

and

$$\hat{H}_{int} = \sum_n E_n(\hat{X}_{Dp}, \hat{P}_{Dp}) \otimes F_n(\hat{\zeta}_{Eq}, \hat{\Pi}_{Eq}) \tag{7}$$

Needless to say, the measurements of e.g. $\hat{x}_{Ai}$, or $\hat{\zeta}_{Dp}$, may be interpreted as the measurements of the observables of the composite system. Yet, the supposed separabilities of the Hamiltonian allow interpretation in terms of the subsystems, again bearing some subtlety.

In general, the measurements of the observables of $\mathcal{A}$ and/or of $\mathcal{B}$ partly reveal, yet *quantum mechanically undetermined* values of the observables of both $\mathcal{D}$ and

$\mathcal{E}$–due to Eq. (5), one may obtain e.g. $[\hat{x}_{Ai}, \hat{\zeta}_{D\alpha}] \neq 0$. As a consequence: the inverse of Eq. (5) can not be used for determining the definite values of the observables of $\mathcal{D}$ and $\mathcal{E}$–in contradistinction with the macroscopic experience. On the other side, only the measurements of $\mathcal{A}$ and $\mathcal{B}$ (of $\mathcal{D}$ and $\mathcal{E}$) do not mutually interfere, referring to the mutually compatible observables. Therefore, the measurements of the observables of the "subsystems" belonging to the different divisions are not mutually independent, while the measurements referring to the observables of the subsystems belonging to the *same division* of the composite system are independent. Needless to say, the later is in agreement with the standard, general procedure we have learnt in the "classical domain." As a consequence, the two possible divisions may refer to the two, mutually complementary, possible entanglements in the system $\mathcal{C}$: $\sum_i c_i |\psi_i\rangle_A \otimes |\chi_i\rangle_B$, and $\sum_j d_j |\Phi_j\rangle_D \otimes |\phi_j\rangle_E$ (compare to (Barnum *et al.*, 2003; Zanardi *et al.*, 2004))–which in the position-representation gives rise to the equality, $\sum_i c_i \psi_i(x_{Ap})\chi_i(\xi_{Bq}) = \sum_j d_j \Phi_j(X_{Dm})\phi_j(\zeta_{En})$.

As long as the composite system may be considered to be *isolated*, the two different divisions as described above seem mutually equivalent for an independent observer. This, however, need not be the case for an open composite system, as discussed in (Dugić *et al.*, 2002).

It is probably obvious: a definition of e.g. subsystem $\mathcal{A}$ makes sense *if and only if* the subsystem $\mathcal{B}$ is *simultaneously* defined. This is both a mathematical consequence of the canonical transformations as well as physically a reasonable notion.

Therefore, the concept of *elementarity* as well as of a *subsystem* are *relative*; as to the later, due to the fact that the real systems are usually open, the relativity of "subsystem" actually means relativity of the basic physical concept of "system."

6. DISCUSSION

The problem of "what is system" naturally stems from the foundations of the macroscopic context of the decoherence theory (Section 3). Actually, the states of a macroscopic (open) system are expected to bear robustness against the influence of the environment–cf. Eq. (1)–that should provide both, a *well-defined* macroscopic *system*–that we here discuss–as well as a classical-like dynamics of the system–that is established by the decoherence theory. On the other side, it seems that, nowadays, the physicists are ready to accept the "undivised universe" on the truly microscopic, and partly on the "mesoscopic" scale. To this end, robustness of states does not fit with the supposed quantum holism on the micro/meso- scale. Nevertheless, the issue of "what is system" is also of interest on these scales from both fundamental physical as well as the information-theoretic point of view (as to the later cf., e.g., (Barnum *et al.*, 2003; Zanardi *et al.*, 2004)). Therefore, our conclusions mainly refer to the macroscopic context of the decoherence theory; investigating their extension towards the fully quantum mechanical scales remains partly an open task of our analysis. To this end, the above distinguished use of the condition of separability may be employed as a (plausible) working hypothesis.

Following the fundamentals of the decoherence theory, we have argued that the condition of separability of the interaction Hamiltonian appears as a criterion for making the "dividing line" between the subsystems of a composite system. As we here emphasize, this approach gives automatically rise to the possibility of defining the subsystems, through a *definition* of the *degrees of freedom* (and their conjugate momenta) that is *based on the condition of separability* of the interaction Hamiltonian. We have also seen that the separability is consistent with our macroscopic experience: e.g. the measurements on $\mathcal{B}$ ($\mathcal{A}$) may be performed *independently* on the measurements on the subsystem $\mathcal{A}$ ($\mathcal{B}$); as a benefit of our considerations, the subsystem $\mathcal{B}$ ($\mathcal{A}$) may be defined *only simultaneously* with the subsystem $\mathcal{A}$ ($\mathcal{B}$). The different divisions of the composite system may bear quantum mechanical complementarity, being mutually exclusive divisions of a composite system. This way, the problem of "what is system" seems to be sharpened, and particularly *reduced* to the following problem: "as to what extent, one may ascribe the physical reality to the different divisions of a composite system?" Needless to say, much remains yet to be done in this respect, and the work in this regard is in progress.

In our brief discussion of the hydrogen atom (HA) in Section 4, we have left a few important notions out. Here, we want to emphasize consistency of the analysis given in Section 4 with the foundations of both, HA theory as well as of the decoherence theory.

Actually, one may wonder if the Coulomb interaction might provide decoherence of the electron states; in other words, one may wonder: bearing the foundations of the decoherence theory (cf. Appendix A) in mind, one may wonder why the electron states do not decohere. Actually, the negative energies of the electron in HA suggest that the Coulomb interaction dominates in the system. Then, according to the decoherence theory (Section 3), one might expect that the electron states might decohere. In answer to this question, we emphasize: the proton is much too small a system in order to play the role of the environment for the electron. In effect, the proton appears as a *source* of the external *field* for the electron–the Coulomb field–not as a dynamical system such as the environment in the decoherence theory. For this reason, the electron remains an isolated system that is subject to the Schrodinger equation, rather than to the decoherence process. The analogous question applies to the composite system $CM + R$. Actually, naively, one might expect that the separable (total) Hamiltonian Eq. (4) should provide the occurrence of decoherence of states of the system R. Again, the answer is very much the same: the system CM is much too small and effectively plays the role of the source of the external field for the system R. Now, we obtain the fully consistent picture: if the proton (or CM) were large enough, the electron (or the system R) states might have decohered. Bearing this in mind, it is clear: we use the HA model as a paradigm for re-defining separability of the (interaction) Hamiltonian, without claiming the occurrence of decoherence in HA. Finally, the analysis of HA gives rise to both, consistency of our considerations as well as their applicability to the *realistic* models.

Once made, a division of a composite system $\mathcal{C}$ may be extended (further "coarse graining" of $\mathcal{C}$) in accordance with the above criteria–e.g. $\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2 + \ldots$. This might also be a basis for certain progress in respect to another fundamental problem of "what is object" (Omnes, 1994). Actually, a collection of the degrees of freedom–defined e.g. according to the method here proposed–does not *per se* define the "object"–that is understood (Omnes, 1994) as a spatial form (a shape) of the system. The concept of "object" comes from the macroscopic sector yet being of significant interest even for certain mesoscopic systems, such as the macromolecules (Raković *et al.*, 2004) (and the references therein).

Finally, our discussion and conclusions are applicable virtually to any complex quantum system. However, its relevance for the realistic systems remains yet to be investigated in the purely quantum-mechanical ("microscopic") as well as in the mesoscopic context. To this end, it is interesting to compare our approach and conclusions with the approach of Zanardi *et al.* (2004) and Barnum *et al.* (2003). A common element of Zanardi *et al.* (2004) and our considerations is the notion on the importance of interaction in the composite system in defining the subsystems. While this is a conclusion in (Zanardi *et al.*, 2004), we still use this notion (stemming from the decoherence theory) in developing the general models of our analysis. The approach of Zanardi *et al.* (2004) is based on certain axioms referring to the "experimentally accessible observables," which is yet an open issue of our approach. Our approach is characterized by the pointing out separability as an operational tool in defining the subsystems. On the other side, rejecting the "reference to a preferred subsystem decomposition" of a composite system, (Barnum *et al.*, 2003) seem essentially to point to the relativity of the concept of "subsystem"–in analogy with our conclusion. However, being an operational analysis of entanglement, their paper does not directly tackle the issue of "what is system." Nevertheless, we believe, that the conclusions of (Barnum *et al.*, 2003; Zanardi *et al.*, 2004) are consistent with our conclusions, which still follow from the foundations of the "environment-induced superselection rules" (or decoherence) theory.

7. CONCLUSION

Decoherence theory is well-suited for addressing the fundamental problem of "what is system." Here, we employ the foundatins of the "environment-induced superselection rules" theory. Actually, we employ the condition of separability of the interaction Hamiltonian as a criterion for putting the "dividing line" between the subsystems of a composite system. On this basis, we point out quantum relativity of the very basic physical concept of "system," thus challenging our classical intuition (e.g. by complementarity of the possible different divisions of a composite system) and posing certain questions that might be of significant interest for the information-theoretic issues (such as e.g. the operational use of quantum entanglement in a bipartite quantum system).

The authors thank the Ministry of Science and Environmental Protection, Serbia, for financial support.

APPENDIX A

A composite system is defined by its Hamiltonian:

$$\hat{H} = \hat{H}_{1\circ} + \hat{H}_{2\circ} + \hat{H}_{int}. \qquad (A.1)$$

If the interaction term $\hat{H}_{int}$ can not be reduced to external field for the open system 1, then neither subsystem can be described by the Schrodinger evolution.

Bearing in mind Eqs. (1) and (2), it is easy to prove that separability of $\hat{H}_{int}$ represents an (effective) necessary condition for the occurrence of decoherence (Dugić, 1997).

Actually, when applied to Eq. (1), the expression Eq. (2) implies diagonalizability:

$$_1\langle m|\hat{H}_{int}|n\rangle_1 = 0, \quad m \neq n \qquad (A.2)$$

where $\{|m\rangle_1\}$ is a common eigenbasis of the observables of the system 1, $\{\hat{B}_{1i}\}$, in Eq. (2).

On the other side, with *restriction* to the almost-periodic-functions formalism (cf. Zurek (1982) and references therein), non-diagonalizability of $\hat{H}_{int}$ in a basis of the system 2, $\{|\alpha\rangle_2\}$, does not give rise to the occurrence of decoherence. Needless to say, a (as yet a hypothetical) more general formalism might challenge our conclusion. Bearing in mind that the exceptions are not known yet (Dugić, 1998), we stem the following rule: diagonalizability of $\hat{H}_{int}$ in a basis $\{|\alpha\rangle_2\}$ represents an effective necessary condition for the occurrence of decoherence.

Altogether, the two diagonalizabilities stem (cf. Definition 4.1) the interaction Hamiltonian separability as the effective necessary condition for the occurrence of decoherence.

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